

The General Solutions of the Robertson-Walker Null-geodesic and their Implications

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The Robertson-Walker metric gives mathematical expression to three widely-held assumptions about the nature of the observable universe. It is shown that the null-geodesic of this metric has little-known solutions for the speed and distance of a light-signal relative to its source. Some implications of these results are considered, the most interesting of these being that only in a universe whose 3-space is Euclidean ($k=0$) will the behaviour of light on the cosmological scale be compatible with our understanding of light behaviour on the local scale. It is shown in an Appendix that the derived implications of the metric are entirely consistent with Special Relativity, its underlying principles and its consequences.

Key words: Cosmology; Special Relativity; Light propagation.

1. The Robertson-Walker Metric

The great wealth of astronomical evidence which we now have allows us to make a number of well-founded and generally accepted assumptions about the large-scale features of our universe. These are:

I. The universe can be considered as an ensemble of fundamental particles (the galaxies and clusters of such) whose distribution is homogeneous from the viewpoint of any fundamental observer, that is, an observer associated with a fundamental particle.

II. The universe is expanding according to Hubble's law, such that the mutual recession speed w between any pair of fundamental particles is proportional to the distance $r(t)$ separating them, so that

$$w = Hr, \quad (1)$$

where r and w are estimated from the luminosity and the red-shift of the spectrum of the light received from distant galaxies. H is the Hubble constant, whose present estimated value is about 60 km/sec per megaparsec; for evolutionary models of the universe H will in general vary with t and so provides a measure of t known as cosmic time.

III. Apart from local irregularities, the appearance of the universe and the laws of nature are the same in all directions from the viewpoint of any fundamental

observer. This assumption is known as the Cosmological Principle and is well-supported by astronomical observations.

These assumptions may be represented mathematically by a general relativistic-type metric named after its separate proponents, H. P. Robertson and A. G. Walker. This representation incorporates a scale factor $R(t)$ corresponding to the nature of the expansion assumed, and also a parameter k corresponding to the assumed space curvature of the model of the universe considered. In view of the assumed isotropy and homogeneity of the model, k is taken as constant on the cosmological scale.

The reference frame, employed by the metric, is based on a set of fundamental particles with associated observers constituting the model, and clearly any one of these, say F_0 , can be taken as the origin of the frame. In terms of spherical polar space coordinates, the Robertson-Walker metric is then given by

$$d\sigma^2 = dt^2 - \frac{R^2(t)}{c^2(1 + \frac{1}{4}kr^2)} [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2], \quad (2)$$

where c is the speed of light, r , θ and ϕ are the fixed comoving (polar) coordinates of any fundamental particle or observer relative to F_0 at the origin of the system, and t is its cosmic time coordinate.

The curvature parameter k may take values of 0, +1, or -1 depending on whether the geometry of the model's 3-space is assumed to be Euclidean, spherical, or hyperbolic, respectively. Since this metric obtains

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equally for all fundamental observers, the interval $d\sigma$ is invariant with respect to all such observers. The case of $d\sigma=0$ corresponds to a null-geodesic, that is, to the path of a light ray in a universe described by (2).

We note that the metric is invariant with respect to the observations of any fundamental observer in accordance with the Cosmological Principle; it defines a fundamental reference frame in respect to which any fundamental particle/observer can be considered as at the origin of the frame, whereby all other particles are distinguished by their comoving coordinates (r, θ, ϕ) or in cartesian by (x, y, z) , etc. Thus we can consider that there exists a number of equivalent sub-fundamental frames, each based on a different fundamental particle as its origin. We note also that the cosmic time variable, t , is a measure of time common to all fundamental observers, being based on the expanding distance between any pair of fundamental particles.

The employment of comoving coordinates means that, in respect to any sub-fundamental frame, the coordinates of all fundamental particles remain unaltered with respect to time, despite the expansion of the system. In other words, the axes of any such subframe are considered to be expanding also, with a lengthening of their units, according to the same (Hubble) Law as applies to the mutual recession of any pair of fundamental particles. Note that, in the context of the metric, the product of the scale factor and the r coordinate, that is $R(t)r$, must have the dimension of a length and, indeed, represents a measure of the increasing distance, r , of a fundamental particle from the origin of a particular subframe.

Hence, we may write

$$r(t) = r R(t) \quad (3)$$

and

$$\dot{r} = r \dot{R}(t), \quad (4)$$

where \dot{r} is the mutual recession speed, denoted by w in (1), between a pair of fundamental particles, one of these being treated as located at the origin of the fundamental frame; however, the above results are quite general since any fundamental particle can be so treated. It will be confirmed below that r can be considered as a luminosity distance, and that w is directly related to the redshift ratio $\Delta\lambda/\lambda$ denoted by z ; however, the $w-z$ relationship depends crucially on the nature of the assumed cosmological model.

It follows from (3) and (4) that

$$\dot{r} \equiv w = (\dot{R}/R) r, \quad (5)$$

which is an expression of Hubble's Law with

$$\dot{R}/R = H.$$

It is seen that, for most models of the universe where R is assumed to be an (increasing) function of time, H (and w) must also vary with time; so that $w(r, t)$ is a function of time as well as of radial distance. However, in the context of our assumptions, particularly the Cosmological Principle and the apparent universality of Hubble's Law in the observable universe, we may allow that at any epoch of cosmic time the value of H is the same in all parts of the (observable) universe; in this sense, H is a universal 'constant'.

We note, finally, that the fundamental frame, defined by the metric (2), is observationally definable in two ways:

(i) Only for an observer 'stationary' with respect to this frame, that is a fundamental observer, will the Hubble Law appear to operate in the same way in all directions; for a 'moving' observer, that is one moving relative to the fundamental frame, the Hubble Law will appear directionally distorted, and the distortion may be employed to deduce the speed and direction of the observer's movement relative to the fundamental frame.

(ii) Following the discovery by Penzias and Wilson, in 1965, of a universal microwave background (of temperature 2.7 K), we can now actually measure our speed and its direction relative to this radiation background and hence relative to the fundamental frame associated with the essential matter and radiation constituting our universe.

2. The Einstein-Friedmann Constraints

The propagation of light in the universe, relative to the frame employed in the Robertson-Walker metric, is described by the metric with $d\sigma=0$; that is

$$dt^2 = \frac{R^2(t)}{c^2(1 + \frac{1}{4} k r^2)^2} [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2]. \quad (6)$$

Many results may be derived from the metric (2) and its null-geodesic (6), as shown, for example, by Heidmann [1], but the solution for distances travelled by light relative to its source is hampered by the problem of finding a physical meaning for the comoving coordinates, particularly for the coordinate r . In the context of the metric, this coordinate is time-invariant and, in general, it must also be considered as dimen-

sionless, since otherwise the denominator term, $(1 + \frac{1}{4} k r^2)$ would not be dimensionally homogeneous except for the case of $k=0$.

However, the metric and its null-geodesic can accommodate non-zero values of k as well as many forms of $R(t)$ depending on the nature of the expansion assumed for the model universe described by the metric. The choice of k and $R(t)$ is constrained, to some extent, by Einstein's field equations whose cosmological implications were derived by Friedmann [2] in the form

$$\dot{R}^2 = 8\pi GQ/3R - kc^2 \quad (7)$$

for a universe for which both the cosmological constant, Λ , and its matter pressure, p (at any point), are assumed to be zero; and where $Q = \rho R^3 = \rho_0 R_0^3 = \text{constant}$, since the (decreasing) density of cosmic matter is inversely proportional to R^3 . The value of Q is normally estimated from an estimate of ρ_0 , the present cosmic matter density ($\sim 10^{-28} \text{ kg m}^{-3}$), and by taking $R_0 = ct_0$, interpreted as the radius (or radius of curvature) of the observable universe at the present time, t_0 .

An important solution of (7) is obtained for $k=0$, that is,

$$R(t) \propto t^{2/3}. \quad (8)$$

These conditions describe a model known as the Einstein-de Sitter universe which has been embraced by most cosmologists as the model most-closely supported by current astronomical observation and physical theory.

The solutions for $k=1$ and $k=-1$ are most conveniently expressed in terms of a parameter, (say) q , thus:

$$\left. \begin{aligned} R &= \alpha(1 - \cos q) \\ ct &= \alpha(q - \sin q) \end{aligned} \right\} \quad \text{for } k=1$$

and

$$\left. \begin{aligned} R &= \alpha(\cosh q - 1) \\ ct &= \alpha(\sinh q - q) \end{aligned} \right\} \quad \text{for } k=-1,$$

where

$$\alpha = 4\pi GQ/3c^2.$$

It is seen that, for both solutions, $t=0$ when $q=0$ (and so is R), and that $\dot{R}=\infty$ suggesting an 'inflationary' expansion near $t=0$. However, of greater interest, it also follows for both solutions that for q and t very small,

$$R \approx \alpha q^2/2 \quad \text{and} \quad ct \approx \alpha q^3/6,$$

so that for both models the expansion develops near $t=0$ according to

$$R \propto t^{2/3},$$

as for the Euclidean ($k=0$) model.

However, as q and t increase, the elliptic space ($k=1$) model has $R(t)$ increasing to a maximum ($=2\alpha$), where $\dot{R}=0$, and then decreasing again, etc., along a cycloid curve. Whereas for the hyperbolic space ($k=-1$) model, $R(t)$ increases indefinitely and develops towards a uniform rate of expansion expressed by

$$R = ct + \gamma, \quad \dot{R} = c,$$

where γ is a constant.

These are the only solutions for p and Λ zero if we assume (as did Einstein) that G is a time-invariant constant. However, if G is not taken as time-invariant then there are two more self-consistent solutions of the Friedmann equation (7) for $k=0$. Thus if we take $G(t) \propto t$, then (7) is satisfied by $R(t) \propto t$, a result depicting a universe with a uniform rate of expansion associated with a cosmological acceleration field [3] which independently gives rise to a value of G proportional to the cosmic time t .

Then there is the Dirac (large numbers hypothesis) model [4] which proposes that

$$G(t) \propto t^{-1}$$

and, independently of Einstein and Friedmann, that $k=0$ and

$$R(t) \propto t^{1/3}.$$

It is seen that this model also satisfies Friedmann's equation for $k=0$. The notion of a slowly-varying G (proportional or inversely proportion to t) is not radically incompatible with Einstein's field equations since G would then still be very close to constant (to one part in 10^{10} over a period of a year), and so would not prejudice the essential gravitational consequences described by the field equations.

3. Solution of the Robertson-Walker Null-geodesic for an Einstein-de Sitter Universe

For this model $k=0$ and $R(t) \propto t^{2/3}$. We may express $R(t)$ in a form which has the dimension of a length, by writing

$$R(t) = ct_0(t/t_0)^{2/3}. \quad (8)$$

where t_0 is any conveniently-chosen instant of cosmic time. Hence it follows from (3) that r must be dimensionless, satisfying the dimensional requirement of the Robertson-Walker metric for all permissible values of k . We note also that

$$r = r/R(t) \quad (9)$$

and that both $R(t)$ and r are distances expanding with time according to $t^{2/3}$, so that r is a pure (ratio) number different, in general, for different fundamental particles. We can determine this number, associated with any such particle, by considering the combination of (8) and (9) at time, $t = t_0$; whereby

$$r = r_0/ct_0,$$

where $r_0 \equiv r(t_0)$ is the distance of the particle from the chosen origin at $t = t_0$. Thus r is directly related to the distance r_0 , and we can also rewrite (9) as

$$r = r_0(t/t_0)^{2/3},$$

an intelligible result, consistent with our interpretation of the various radial measures employed.

Now consider a light-signal emitted at cosmic time, t_0 , from a fundamental particle, F_0 , whose location is taken as the origin of the frame. The equation of the signal's radial light-path must satisfy the null-geodesic (6), with $k=0$, so that it reduces to

$$\pm c \, dt = R(t) \, dr.$$

The minus sign would apply to a light-signal approaching a point taken as the origin of the system; so for our case, the path is sufficiently described by

$$c = R(t) \dot{r}. \quad (10)$$

The signal will pass successive fundamental particles at increasing distances from F_0 . When it reaches the fundamental particle/observer F (say) at cosmic time, t , the distance, s , from its source is identical to the radial distance, r , of F from F_0 ; that is

$$s \equiv r = rR(t).$$

It will be convenient to introduce and employ $s(t)$ for the distance of the signal from its origin at time, t , since although s and r are formally equal, they have different meaning and different time-derivatives. Thus \dot{r} ($=w$) is given by (4), in which r is a time-invariant number associated with F ; but the speed of light, \dot{s} , relative to its origin, F_0 , is given by

$$\dot{s} = r\dot{R}(t) + \dot{r}R(t), \quad (11)$$

since the light-signal's r -coordinate also varies as the light passes a succession of fundamental particles. Finally, the light-path must satisfy (10), so that (11) becomes, on invoking (9) and then (5),

$$\begin{aligned} \dot{s} &= r\dot{R}/R + c, \\ &= c + w(r, t), \end{aligned} \quad (12)$$

signifying that the light-signal passes each successive fundamental particle at the same speed c . For the Einstein-de Sitter model, $R(t)$ is given by (8), so that we can also write

$$\dot{s} = c + 2s/3t. \quad (13)$$

Integrating, and using the initial conditions that $s=0$ when $t=t_0$, we obtain

$$s = 3ct[1 - (t_0/t)^{1/3}]. \quad (14)$$

Also, on account of (5) and (8), again

$$w = 2s/3t = 2c[1 - (t_0/t)^{1/3}]. \quad (15)$$

Now, it also follows directly [1] from the geodesic that the spectral ratio, λ/λ_0 [where λ_0 is the wavelength of the signal at $F_0(t_0)$ and λ its wavelength at $F(t)$], is related to t and t_0 by

$$\lambda/\lambda_0 = R(t)/R(t_0). \quad (16)$$

Hence for the Einstein-de Sitter universe, we obtain

$$\begin{aligned} \lambda/\lambda_0 &= (t/t_0)^{2/3} \\ &= 1/(1 - w/2c)^2, \end{aligned} \quad (17)$$

on invoking (15).

The above results might be considered as rather startling, implying as they do that the observational horizon for this model is $3ct$ (at the present epoch, t , of cosmic time), and that galaxies are receding at a speed close to $w=2c$ near this horizon. However, the results are a valid consequence of the Robertson-Walker metric description of a universe governed by the assumptions and definitions normally accepted by cosmologists. The results are derivable from the geodesic even without the introduction of the variable, s ; the essential result (14) follows by direct integration from the geodesic expression (10) with $R(t)$ as given by (8), employing the initial condition that $r=0$ when $t=t_0$, and then substituting for r according to (9). The solution for r is, in fact, presented by Heidmann ([1], p. 108). Many further consequences, including important results pertaining to astronomical observation,

flow from the above, as shown in a comprehensive monograph by Paparodopoulos [5].

4. The Nature of an Einstein-de Sitter Null-geodesic

Probably the most startling and (for some) disturbing result is the one pertaining to the speed of light relative to its source as given by the result (12). This result *appears to* directly contradict two of the most sacrosanct tenets of modern physics – that the speed of light is constant, and that it is equal to c with respect to every inertial frame. But the contradiction is only *apparent* as will be shown below and, further, the result is eminently consistent with other well-known conclusions about the behaviour of light.

We note, first, that the ‘stretching’ of the light-(speed), according to (12), as it propagates in an expanding universe, offers an immediate and intelligible interpretation of the red-shift which such light develops. The result (16) – a direct consequence of the Robertson-Walker metric – depicts the wave-length expanding in the same proportion as the expanding distance between any pair of fundamental particles; so that the wave-length expansion can be considered as directly relating to the corresponding ‘stretching’ of the speed of the light *relative to its source*. Such an interpretation of the cosmological redshift phenomenon was first proposed by McCrea [6] on the basis of a cosmological light-speed equation similar to the result (12) but for a light-signal approaching the (chosen) origin of the fundamental frame. He observes that “(This) ensures that the light-speed relative to the local fundamental observer is always the universal constant c ”, so that this gives the same law of light-speed for every sub-frame of the fundamental frame as implied by the Cosmological Principle. Hence we may call (12) McCrea’s hypothesis but, in fact, it is a direct consequence of the Robertson-Walker metric when applied to the Einstein-de Sitter model, and so it stands or falls according to the validity of the model, the metric and its underlying assumptions.

One might ask: What ‘causes’ the light-speed and its wave-length to stretch in the manner described by (12) and (16)? – the same as ‘causes’ the mutual recession of any pair of fundamental particles? Clearly these two cosmological phenomena are closely linked. Our expanding universe is the unique arena for all cosmological behaviour of matter and energy which, as we know, interact closely at all levels, including the

astral and galactic levels where gravitational fields affect the direction, speed and spectrum of radiation.

We might therefore expect that the propagation of radiation on the cosmological scale would take place *in respect to* the expanding distribution of matter in the universe, that the behaviour of radiation and matter are not independent on the cosmological level either. It may be that both the matter expansion and the light-speed stretching are due to a cosmological acceleration field as described in [3, 7]; however, irrespective of the nature and origin of these apparent phenomena, they are imbedded in the Robertson-Walker metric and are strongly supported by observation. Perhaps the strongest support comes from a maxim of physics which holds that ‘light does not overtake light’. Experience and observation suggest that light from all astronomical sources, whether receding, approaching or relatively stationary, reaches us at the same speed – the same speed as light from local sources, receding, approaching or relatively stationary. This is true for galactic sources receding at $100,000 \text{ km s}^{-1}$ or from quasar sources receding even faster from us. This result is, of course, perfectly consistent with McCrea’s hypothesis and rendered intelligible by it.

The hypothesis *and* the Robertson-Walker metric imply the existence of a fundamental reference frame which is the unique arena for the propagation of light. This explains the basis of the maxim (embraced by Einstein in his Light Principle) that ‘the speed of light is independent of the velocity of its source’ for, in our context, the propagation of light according to (12) takes place with respect to the (hypothetical) fundamental particle at any given locality; it takes place at speed, c , relative to each fundamental particle in its path. More generally, light (and all forms of radiation and energy) propagates at speed, c , relative to the fundamental reference frame. This means that if light is emitted by a source which is not a fundamental particle (which will generally be the case, since most, if not all, cosmic entities appear to have their own proper velocities relative to the (hypothetical) fundamental particle in their locality), its emission speed will be c , in all directions, relative to the (hypothetical) fundamental particle in the locality of the source. In our context, the speed of light is truly independent of the velocity of its source; it is a speed relative to the fundamental frame. Depending on the proper velocity of the source, the emitted light will also be affected by a ‘substratum’ Doppler shift in its spectrum; for very

distant sources, this shift will be cloaked by the much larger cosmological redshift effect, but for nearby sources the substratum Doppler shift may be dominant or comparable with the cosmological effect.

Note that the spectral ratio formula (17) relates the present recession speed of the source, *treated as a fundamental particle*, with a spectral ratio as observed by a fundamental observer. If source and/or observer are not fundamental, then corrections need to be made for the appropriate substratum Doppler effects operating. However, the latter will be negligible for distant galaxies whose recession speeds would dominate any normal proper speeds.

Similarly, the formula (14) relates the distance of a light-signal, observed at time t , from a fundamental source from which it was emitted at time t_0 . Thus s in (14) gives us the distance of a (fundamental) source at the time of observation, and since the observed luminosity depends on the distance travelled *relative* to the source, s can be considered as a luminosity distance offering an estimate of the *present* distance of the source, provided it is fundamental. However, if the source is not a fundamental particle, then again a correction is necessary, remembering that such correction will be very small since proper speeds are normally very small compared to the speed of light.

Finally, we must deal with the most serious objection. The Robertson-Walker null-geodesic defines and deals with an (expanding) fundamental reference frame with respect to which light propagates at speed c . This frame may be shown [3] to be inertial in-so-far as any body which is stationary in this frame (such as a fundamental particle) will remain so; and any body with velocity, u , will continue to pass every fundamental particle in its path at the same velocity, in the absence of external forces. However, the existence of such a privileged frame appears to be in direct contradiction to Special Relativity and its underlying assumptions.

Contrary to appearances, this result is by no means incompatible with Einstein's Special Theory. In fact, it offers a physical interpretation of the Theory, its assumptions and its consequences in a manner which makes the Theory physically intelligible and free of any ambiguities – all this in terms of a *single assumption* flowing from modern cosmology [3]. However, since this paper is concerned with the cosmological implications of the Robertson-Walker metric, we will postpone its relativistic consequences to an Appendix below.

5. Light-Speed Results for other Models

The derivation of the distance travelled by light, its speed (relative to its source), the spectral ratio formula, the observational horizon, etc., from the Robertson-Walker null-geodesic follows for other Friedmann and pseudo-Friedmann models of the universe in the same way as for the Einstein-de Sitter model. However, for our present purposes we will focus on the light-speed result for each of these models.

The Elliptic Space Model

Case of $k=1$, $R(t)$ starts with $R(t) \propto t^{2/3}$ and tends to $R(t) \propto t^0$ during the expansionary half-cycle. In terms of this elliptic space model, and considering the present value of cosmic time, we will describe the present state of expansion by $R(t) \propto t^{1/2}$ as a basis for calculation.

For a light-signal emitted at cosmic time, t_0 , from a fundamental particle, F_0 , whose location is taken as the origin of the frame, the geodesic (6) here reduces to

$$c(1 + r^2/4) = R \dot{r}, \quad (18)$$

where

$$R(t) = c t_0 (t/t_0)^{1/2},$$

and

$$\dot{R}(t) = \frac{1}{2} c (t_0/t)^{1/2}. \quad (19)$$

As before, the distance, s , of the light-signal from its source, F_0 , at time t is equal to the distance, r , of the fundamental particle, F , which it has reached at time t ; so that

$$s \equiv r = r R(t)$$

and

$$\dot{s} = r \dot{R} + \dot{r} R. \quad (20)$$

Also, at time t , F is receding at speed $w (\equiv \dot{r})$ from F_0 , where

$$w \equiv \dot{r} = r \dot{R}. \quad (21)$$

So, combining (18) and (20), we obtain

$$\begin{aligned} \dot{s} &= r \dot{R} + c(1 + r^2/4) \\ &= w + c(1 + w^2/4 \dot{R}^2), \quad \text{using (21),} \end{aligned} \quad (22)$$

$$= c + w + (w/c)(t/t_0)w, \quad \text{using (19).} \quad (23)$$

It is seen that for this model, light still travels with respect to the fundamental frame but passes successive fundamental particles at speeds increasingly greater than c ! This would mean that light from galaxies at

different distances (from us) would reach us at different speeds. For example, light from a galaxy, 200 Mpc distant and hence receding from us at about $12,000 \text{ km s}^{-1}$, would reach us at a speed of at least 480 km s^{-1} faster than light from nearby galaxies (in the same direction) or from a local source; and this ignores the (t/t_0) factor which for this example would be about 1.04. Galaxies at such a distance and with a spectral ratio of about 1.02 (for this model) are considered as medium-distance galaxies, and many such are observed and their light analysed, so that the violation of the 'light does not overtake light' maxim and of McCrea's hypothesis makes this model quite untenable.

Further, the spectral formula for this model is given by

$$(\lambda_0/\lambda) \tan [(\lambda/\lambda_0) - 1] = w/c,$$

whereby the theoretical maximum value of the spectral ratio would be $(\pi + 2)/2 \sim 2.57$. Again this violates observation: spectral ratios of up to 5 and over are commonplace, particularly for light received from distant quasars.

Hyperbolic Space Model

$k = -1$, $R(t)$ starts with $R(t) \propto t^{2/3}$ and tends towards $R(t) \propto t^1$. In view of the time which has elapsed since the (hypothetical) beginning of the apparent expansion of the universe, we may assume that the scale-factor is close to $R(t) \propto t$ at the present time, for the purposes of our calculations.

For a light-signal emitted at cosmic time, t_0 , etc. (as previously), we obtain for this case, corresponding to the result (22),

$$\dot{s} = r\dot{R} + c(1 - r^2/4), \quad (24)$$

where here

$$R = ct_0(t/t_0) = ct,$$

so that

$$\dot{R} = c,$$

$$r = r/R = r/ct = w/c$$

and

$$w = \dot{r} = r\dot{R} = rc.$$

Using these results in (24), we obtain

$$\dot{s} = c + w - (w/4c)w.$$

So this model also violates the requirement that light from all sources, receding or not, should reach us at the same speed as light from any local source. For this case, light from a source 200 Mpc distant would reach us at a speed of about 120 km s^{-1} slower than light from nearby sources or local sources in the same direction. The discrepancy is smaller (and in the opposite direction) than for the elliptic space model, but it is certainly significant. Furthermore, the discrepancy increases with the square of w for increasing distances.

The spectral ratio formula, for this case is given by

$$\lambda/\lambda_0 = (1 + w/2c)/(1 - w/2c),$$

so that the ratio is theoretically boundless and associated with an observational horizon where $w = 2c$.

The Pseudo-Friedmann Models with $k=0$

The Dirac model for which $R(t) \propto t^{1/3}$, and the $G(t) \propto t^{-1}$ model.

Looking back at the Einstein-de Sitter model, it is seen that the result (12):

$$\dot{s} = c + w(r, t),$$

follows from the null-geodesic irrespective of the nature of $R(t)$; the result is independent of the assumed nature of the expansion – it depends only on the assumption that the 3-space of the model is taken as Euclidean. Hence, (12) applies to any Euclidean space model.

For the Dirac model the spectral ratio formula is given by

$$\lambda/\lambda_0 = (1 - 2w/c)^{-1/2},$$

so the ratio is here boundless and associated with an observational horizon where $w = c/2$.

For the uniformly-expanding model the spectral ratio formula is

$$\lambda/\lambda_0 = \exp(w/c),$$

so that both the ratio and w are boundless here. This was the first model [8] for which the light-speed result, etc., were deduced from the Robertson-Walker null-geodesic. The deduction follows easily for this case by associating the comoving coordinate, r , with the constant recession speed of each fundamental particle from the chosen origin. The generalisations of this deduction for other models were developed by Paparodopoulos [5, 9].

6. Conclusions and Exclusions

It is seen that a careful consideration of the dimensional requirements in the Robertson-Walker metric makes possible the deduction of precise and astronomically-important formulae for the distance and speed of a light-signal with respect to its source, considered as a fundamental particle, the results depending, of course, on the model assumed. It is surprising that these straight-forward deductions are absent in nearly all cosmological texts and articles, and that they are ignored even when they are presented at conferences and in publications. This may be due to the problem of assigning a physical meaning to the co-moving coordinates, but possibly also to the apparently alarming results which are implied by the deductions. The implication that there exists a 'fundamental reference frame', that light stretches (– accelerates!) on its cosmic travels, that galaxies may be receding from us at speeds greater than $300,000 \text{ km s}^{-1}$ (but this without prejudice to the notion that the speed of light is a limiting speed at any locality of the universe), that the light from such galaxies may reach us (and so render them visible) at a speed depending on the assumed model, and that light may reach us from galaxies at distances beyond the generally assumed horizon equal to ct_0 , where t_0 is the assumed 'age' of the universe at present – these suggestions are anathema to most physicists because they appear as utterly contrary to the behaviour of light on the *local level*. Yet, as we have argued, these notions do not contradict the known behaviour of light on Earth – indeed they reinforce and give added meaning to the precepts that 'light does not overtake light' even if we compare light from a distant receding source with local light, and that 'the speed of light is independent of the velocity of its source'.

In 1905 the existence of a "privileged reference frame" was unknown and unsuspected. Hence Einstein developed his theory without it on the basis only of two empirical principles – his relativity and light principles. However modern astronomy has revealed the existence of a unique and observably-definable fundamental reference frame associated with the large-scale distribution of matter in our universe. Clearly modern physics needs to come to terms with the existence of such a frame and, indeed, as originally suggested by Builder [10] and as will be shown below, the notion of such a frame actually enhances our understanding of Special Relativity and its conse-

quences. This result is by no means *immediately* obvious, so it is still generally considered highly suspect. However, in recent years its validity and advantages have been independently recognised by a growing number of physicists including Bell [11], Selleri [12], and Oliver [13].

The results presented above for the different Friedmann universe models have their own intrinsic interest. It is seen that *only* for the models embracing a Euclidean 3-space (so that $k=0$) does light propagate strictly according to McCrea's hypothesis, thereby satisfying the local experience that light does not overtake light. Elliptic 3-space (with $k=1$) seems to enhance the light-speed beyond c as it passes successive fundamental particles; hyperbolic 3-space diminishes this speed in respect to successive fundamental particles. The latter two results are clearly at odds with observation and suggests that only a universe whose 3-space is Euclidean is compatible with our physical and astronomical experience of light propagation. However, since $R(t)$ varies with time for $k = \pm 1$, they should be looked at more closely.

Appendix: The Consequences of Movement Relative to the Fundamental Frame

The existence of a fundamental reference frame for light propagation means that the speed of light is precisely isotropic with respect to any body (or fundamental particle or associated fundamental observer) which is stationary in respect to this frame, I . It follows, then, that the speed of light will not be isotropic with respect to a body or observer (or their associated reference frame) moving, with velocity u (say), relative to I . It is easily seen that, in respect to such a body, the speed of light approaching the body will be as shown in Fig. 1, where

$$c' = (c^2 - u^2 \sin^2 \theta)^{1/2} + u \cos \theta \quad (25)$$

for the direction making an angle θ with the direction of u . We may call the result (25) the *primary anisotropy effect* due to motion relative of I .

A second consequence of such movement results from the retarded potential effect on the fields (gravitational and/or electromagnetic) associated with the moving body and its constituent particles. Following Einstein's view of gravitational fields and in the context of the existence of a frame such as I , the variation of the potential at any point of a gravitational field, due to the movement of its source, will involve a time-

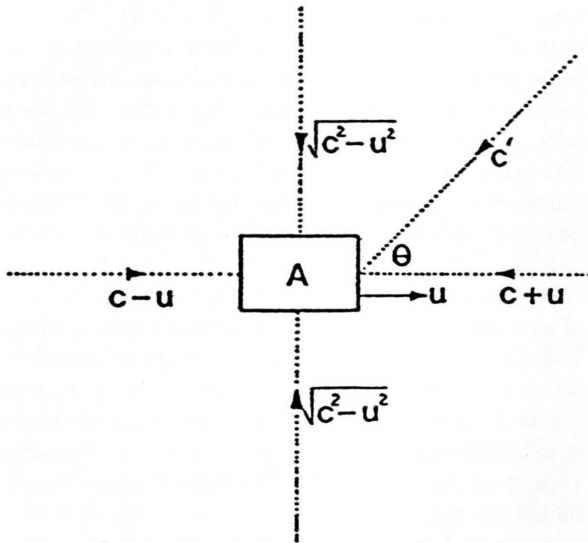


Fig. 1. Speed of light relative to a body moving with velocity u with respect to the fundamental frame, I .

lag depending on the distance of the point from the source, exactly as is well-known for the electromagnetic field of a moving body. It follows [3] that the usual inverse-square law for such a field is then modified by a factor involving the body's speed u relative to I , so that

$$F = \frac{-GmM}{r^2} \frac{(1-u^2/c^2)}{[1-(u^2/c^2)\sin^2\theta]^{3/2}} \quad (26)$$

for any direction making an angle θ with the direction of u , and where m is the rest-mass of a test particle, stationary in I ¹.

The result (26) implies that the Coulomb and gravitational fields (and any other field of this type), linking a moving system of particles, will be correspondingly asymmetric, so that the system must contract in the direction of motion by the factor, $(1-u^2/c^2)^{1/2}$, in order to maintain its internal equilibrium. Hence, a rod of rest-length l inclined at an angle θ to the direction of its velocity u , relative to I , will assume a length l' (according to I observers), where

$$l' = \frac{l(1-u^2/c^2)^{1/2}}{[1-(u^2/c^2)\sin^2\theta]^{1/2}}. \quad (27)$$

¹ The effect on a moving test particle involves only a component of the local field effect. Thus in respect to a comoving particle the effect is diminished by the factor $[1-(u^2/c^2)\sin^2\theta]^{1/2}$ depending on the angle θ made by u with the direction of the field effect at the locality of the particle.

The time-dilation effect now follows as a direct consequence of the interaction of the primary anisotropy and contraction effects. Following Builder [10], consider a light-clock consisting of a rod of rest-length l with a mirror at each end to reflect a beam of light to and fro along the length of the rod. Let the unit of time be taken as the interval between successive light reflections on one of the mirrors which is connected to a photon-counter. When the rod is stationary in I , the unit of time \hat{t} , is given by

$$\hat{t} = 2l/c.$$

However, when such a clock moves with velocity u relative to I , the speed of light relative to the moving rod will, in general, be different for the two directions depending on the orientation of the rod-clock relative to the direction of u . For an angle θ to the direction of u , the two light-speeds, c_1 and c_2 , are given by (25), so that

$$c_1 = (c^2 - u^2 \sin^2 \theta)^{1/2} + u \cos \theta,$$

$$c_2 = (c^2 - u^2 \sin^2 \theta)^{1/2} - u \cos \theta.$$

Hence, the unit of time, \hat{t}' , for the moving rod-clock is given by

$$\hat{t}' = (l'/c_1) + (l'/c_2) = (2l/c)(1-u^2/c^2)^{-1/2} = \beta \hat{t} \quad (28)$$

invoking (27) to relate l' and l . It is seen that the time-dilation result is independent of the orientation of the light-clock, and that it must also apply to all phenomena involving electromagnetic impulses and energy exchanges. It may be shown that other types of clocks will be similarly affected on account of the anisotropy consequences (25), (26), and (27). Hence Builder contended that this effect will manifest itself not only in physical clocks, but in all natural phenomena, both physical and biological, which have an electromagnetic basis.

It is well known that the contraction effect (27) is sufficient to conceal the light-speed anisotropy to an observer moving uniformly relative to I : hence the null-result of all Michelson-Morley type experiments. It follows that, in consequence of the time-dilation effect (28), such an observer will further find that his *measure* of the speed of light, with respect to his co-moving reference frame, is precisely c in all directions – as it is for (fundamental) observers stationary in I . Thus Einstein's Light Principle becomes physically intelligible in our context.

It was in accordance with this Principle that Einstein specified light-signal measurement conventions (which assume light-isotropy) for synchronising clocks and estimating the space and time coordinates of distant events, etc. However, it now becomes clear that if observers, associated with a 'moving' reference frame, treat their internal system² as 'stationary' and so synchronise their clocks according to Einstein, then these clocks are bound to appear non-synchronous to an observer in a different frame, and vice-versa. In effect, the employment of Einstein's convention produces [3] a synchronism discrepancy effect given by

$$(\beta u d/c^2) \cos \theta, \quad (29)$$

where d is the I measure of the length-interval separating the (moving) Einstein-synchronous clocks in question, and θ is, as usual, the angle made by this interval with the direction of \mathbf{u} . The result is, of course, equivalent to the relativity of the simultaneity factor deduced by Einstein from the Lorentz transformation, and since it is a function of \mathbf{u} it leads [3] to the relativity of simultaneity in respect to *any* pair of inertial frames in relative motion and hence having different velocities with respect to I . It is seen that these results demystify Einstein's Light Principle, time dilation and the "relativity of simultaneity".

The anisotropy consequences, (25)–(29), which affect moving bodies and the observations of moving observers, provide a complete physical interpretation,

² Given that the fundamental frame, I , is an inertial system, in the sense as described above, then any frame in uniform motion relative to I at any locality must also be an inertial system.

free of any ambiguity, of Special Relativity. Their interaction is expressed by the Lorentz transformation. They show how any why the Light Principle operates in respect to all inertial frames, they explain (cf. [14]) why any local experiment designed to detect an absolute velocity is bound to yield a null-effect. The existence of a fundamental reference frame provides a physical basis for these absolute anisotropy effects, and their interaction produces the *local* observational equivalence of all inertial frames in respect to the laws of nature as expressed by the Lorentz transformation. This latter result, a manifestation of the principle of relativity, is by no means fortuitous: it is a consequence of a widely-operating, action-reaction principle proclaimed by Newton, where in this case the anisotropy reactions to uniform motion, relative to I , nullify precisely the observation by a comoving observer of such motion. Only by astronomical observation can we discern the existence of the fundamental frame and our movement relative to it.

The interpretation, as above, of Special Relativity completes Lorentz's programme for such an interpretation. Lorentz's developing concept of an aether converged towards the notion that it had only the single light-propagation property of our fundamental frame. He was aware of the Heaviside retarded-potential effect which must produce the length-contraction result, but remained to the end rather confused about the nature of time-dilation and about the relativity of simultaneity result which is a direct intelligible consequence of Einstein's measurement conventions. It remained for Builder [10] to disclose the source of these two separate time results and hence present a fully-integrated "Neo-Lorentzian" interpretation of Einstein's Special Theory.

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